Summations

Review Material for Algorithm Analysis

Definition

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + ... + f(b)$$

$$\sum_{i=1}^{5} i = ?$$

$a = 1$

$b = 5$

$f(i) = i$

$$\sum_{i=7}^{11} ? = 3 + 3 + 3 + 3 + 3$$

$a = 7$

$b = 11$

$f(i) = ...$

Summations

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Definition

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + ... + f(b)$$

$$\sum_{i=3}^{6} ? = 4 + 5 + 6 + 7$$

$a = 3$

$b = 6$

$f(i) = ...$

$$= \sum_{i=4}^{7} i$$

$$\sum_{i=-2}^{2} ? = 4 + 2 + 2 + 4$$

$a = -2$

$b = 2$

$f(i) = ...$

Connection Between Loops and Summations

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

```
void loop1() {
    int i, j;

    for ( i = 1; i <= 5; i++ )
        j = i;
}
```

Is this it?

Connection Between Loops and Summations

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

```
void loop3() {
    int i, j, k;

    for ( i = 1; i <= 5; i++ )
        for ( j = 0; j < i; j++ )
            k = i+j;
}
```

Is this it?

Make sure to mark the right one on the sheets...

Connection Between Loops and Summations

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

```
void loop2() {
    int i, int sum = 0;

    for ( i = 1; i <= 5; i++ )
        sum = sum + i;
}
```

Is this it?

So, What Was the Summation For ‘`loop2`’?

```
void loop2() {
    int i, int sum = 0;

    for ( i = 1; i <= 5; i++ )
        sum = sum + i;
}
```

Summation Form vs Closed Form

$$\sum_{i=0}^{N} i = 1 + 2 + 3 + ... + (N-3) + (N-2) + (N-1) + N$$
$$= (1+N) + \boxed{2 + 3 + ... + (N\text{-}3) + (N\text{-}2) + (N\text{-}1)}$$
$$= (1+N) + \boxed{2 + 3 + ... + (N\text{-}3) + (N\text{-}2) + (N\text{-}1)}$$
$$= (1+N) + (1+N) + \boxed{3 + ... + (N\text{-}3) + (N\text{-}2)}$$
$$= (1+N) + (1+N) + \boxed{3 + ... + (N\text{-}3) + (N\text{-}2)}$$
$$= (1+N) + (1+N) + (1+N) + \boxed{... + (N\text{-}3)}$$
$$= ...$$
$$= ...$$

Series/Sums

- Arithmetic Series:
  - An arithmetic series is a series of values where value i is value $i-1$ plus something. For example 1, 3, 5, 7, 9, ...
  - An arithmetic sum is the sum of an arithmetic series of values. For example 1 + 3 + 5 + 7 + 9 + ...
- Geometric Series:
  - A geometric series is a series of values where value i is value $i-1$ times something. For example 2, 4, 8, 16, 32, ...
  - A geometric sum is the sum of a geometric series of values. For example 2 + 4 + 8 + 16 + 32 + ...
- We encounter arithmetic and geometric sum/series often in estimating the cost of an algorithm

Summation Form vs Closed Form

So, for the shown example, $\frac{N}{2}(1+N)$ is the closed form of $\sum_{i=0}^{N} i$

- What is the point of the summation form?

...

- What is the point of the closed form?

...

- Does the summation not provide an answer?

...

What We Will Often Use

$$\sum_{i=0}^{\infty} A^i = S, \text{ assuming } 0 < A < 1$$

$$S = 1 + A + A^2 + A^3 + ...$$
$$AS = A + A^2 + A^3 + A^4 ...$$
$$S - AS = (1 + A + A^2 + A^3 + ...) - (A + A^2 + A^3 + A^4 ...)$$
$$= ...$$
$$S(1-A) = ...$$
$$S = ...$$

What We Will Often Use

$$\sum_{i=0}^{N} A^i = \frac{A^{N+1}-1}{A-1}, \text{ with no assumptions on } A$$

$$\sum_{i=0}^{N} 2^i = 2^{N+1}-1, \text{ careful, this is just a special case}$$

$$\sum_{i=0}^{N} i^2 = \frac{N(N+1)(2N+1)}{6} \approxeq \frac{N^3}{2}$$

$$\sum_{i=0}^{N} i^k \approxeq \frac{N^{k+1}}{|k+1|}, \text{ provided } k \neq -1$$

And A Few Problems

$$\sum_{i=10}^{20} i, \ \sum_{i=1}^{20} 5i, \ \sum_{i=1}^{20} k, \ \sum_{i=a}^{b} k, \ \sum_{i=1}^{20}\sum_{j=1}^{10} 2, \ \sum_{i=1}^{20}\sum_{j=1}^{10} j, \ \sum_{i=1}^{20}\sum_{j=1}^{10} (i+j)$$